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Hydromagnetic Convective Heat and Mass Transfer Flow past a Vertical Wavy Wall with Variable Viscosity, Thermal Conductivity, Thermal Radiation, Chemical Reaction and Thermophoresis

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ABSTRACT

The effect of variable viscosity, thermal conductivity, thermophoresis on convective heat transfer flow past a wavy surface with heat sources and thermal radiation is considered, the non-linear, coupled equations have been solved by employing Runge-Kutta shooting method. We find that an increase in radiation parameter (Rd), thermal conductivity parameter (β), thermophoretic parameter (τ) and amplitude parameter (a) reduces the velocity, enhances the temperature and concentration. Also skin friction, Nusselt and Sherwood numbers reduces with increasing amplitude (a) of wavy surface.

Key Words: *Thermophoresis, Heat source, Magnetic field, Variable viscosity, Thermal conductivity, amplitude of wavy surface.*

I. INTRODUCTION

With the fuel crisis deepening all over the world, there is a great concern to utilize the enormous power beneath the earth's crust in the geothermal region. Liquid in the geothermal region is an electrically conducting liquid because of high temperature. Hence the study of interaction of the geomagnetic field with the fluid in the geothermal region is of great interest, thus leading to interest in the study of MHD convection flows through porous medium.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. The volumetric heat generation has been assumed to be constant or a function of space variable [4, 5].

The effect of the thermophoresis is so widespread in many practical applications in removing small particles from gas streams, in studying particles material deposition on turbine blades and in determining exhaust gas particle trajectories from combustion devices. Several authors, Mahdy and Hady [6], Postelnicu [8], Tsai and Huang [10] have investigated the effect of thermophoresis in vertical plate, micro-channel, horizontal plate and parallel plate.

In light of these applications, the effect of thermophoresis in laminar flow of a viscous incompressible fluid was first established by Goren[3].Wu and Grief[12] studied the thermophoresis deposits including an application to the outside vapour deposits process. Sreenivasa Reddy et el[9] and Aliveni et al[1]. But Wang and Chen[11] was the first investigated thermophoresis deposition of particles from a boundary layer flow onto a continuously moving wavy surface without considering the Darcy porous medium. Mallikarjuna [7] have discussed the effect of thermophoresis on convective heat and mass transfer flow over a vertical wavy suface in a porous medium with variable properties.

II. FORMULATION OF THE PROBLEM

We analyse steady, incompressible, two-dimensional laminar natural convective heat and mass transfer flow over a vertical wavy surface embedded in a saturated porous medium. The Darcy law is used to describe the fluid saturated porous medium.The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium.The wavy surface profile is given by

$$y=\bar{\sigma}(\bar{x})=\bar{a}Sin(\frac{\pi\bar{x}}{l}) \qquad (1)$$

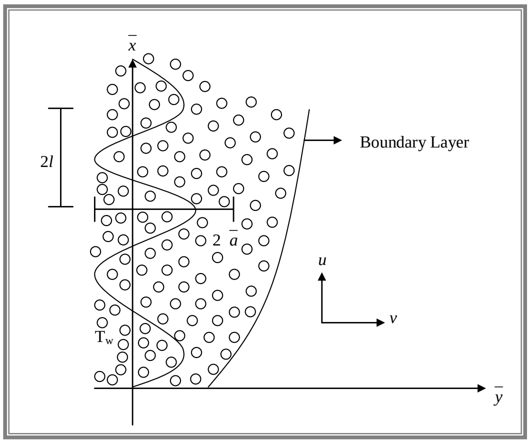


Fig. – 1 : Physical Configuration and Co-ordinate System

where l is the characteristic length of wavy surface and $\bar{a}$ is the amplitude of the wavy surface.The wavy surface is maintained at constant temperature Tw which are higher than the ambient fluid temperature T_∞. We consider the natural convection-radiation flow in the presence of heat sources to be governed by the following equations under Boussinesq approximations:

$$\frac{\partial}{\partial \bar{y}}(\frac{\mu}{k}\bar{u})-(\sigma\mu_e^2 H_o^2)\frac{\partial u}{\partial y}=\frac{\partial}{\partial \bar{x}}(\frac{\mu}{k}\bar{v})+\rho g(\beta_o\frac{\partial T}{\partial y}+\beta_1 T\frac{\partial T}{\partial y}+\beta_c\frac{\partial C}{\partial y}) \quad (2)$$

$$\vec{u}\frac{\partial T}{\partial y}+\vec{v}\frac{\partial T}{\partial \vec{y}}=\frac{\partial}{\partial \vec{x}}(\alpha\frac{\partial T}{\partial \vec{x}})+\frac{\partial}{\partial \vec{y}}(\alpha\frac{\partial T}{\partial \vec{y}})+\frac{DK_T}{C_sC_p}(\frac{\partial^2 C}{\partial \vec{x}^2}+\frac{\partial^2 C}{\partial \vec{y}^2})+\frac{1}{C_p}Q_H-\frac{1}{C_p}\nabla..q_r \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})-kc(C-C_\infty)+\frac{D_BK_T}{T_m}(\frac{\partial^2 T}{\partial \vec{x}^2}+\frac{\partial^2 T}{\partial \vec{y}^2})-\frac{\partial}{\partial x}(U_TC)-\frac{\partial}{\partial y}(U_TC) \quad (4)$$

The relevant boundary conditions are

$$\vec{u}=0,\bar{v}=0,T=T_w\ ,C=C_w\ \ at\ \ \bar{y}=\bar{\sigma}(\bar{x})=\bar{a}\ Sin(\frac{\pi\bar{x}}{l})$$

$$\vec{u}\to 0,T\to T_\infty\ ,C\to C_\infty\ \ as\ \ \bar{y}\to\infty \quad (5)$$

where $\bar{u}$ and $\bar{v}$ are the volume averaged velocity components in the directions of x and y respectively, T, C are temperature, Concentration respectively, ρ is the density of the fluid, μ is the dynamic viscosity of the fluid,k is the permeability of the porous medium, σ is the electrical conductivity, μ_e is the magnetic permeability, H_0 is the strength of the magnetic field, D_B is the molecular diffusivity,kc is the coefficient of chemical reaction, β_o are the coefficients of thermal expansion, α is the thermal conductivity, q_r is the radiative heat flux, g is the acceleration due to gravity and Q_H is the strength of the heat source. K_T is the thermal diffusion ratio, T_m is the mean fluid temperature, U_T and V_T are thermophoretic velocities which can be written [12]

$$U_T=-\frac{k\nu}{T_r}\frac{\partial T}{\partial x}\quad \text{and}\quad V_T=-\frac{k\nu}{T_r}\frac{\partial T}{\partial y} \quad (6)$$

$$k_{nf}=\frac{2C_s(\lambda s/\lambda p+C_1k_n)[1+k_n(C_1+C_2e^{-C_3k_n})]}{(1+3C_mk_n)(1+2\lambda_g/\lambda_p+2C_1k_n)}$$

where C_1, C_2, C_3, C_m, Cs, are constants, λ, g and λ_p are thermal conductivities of fluid and diffused particles,respectively and k_n is the Knudsen number where k is thermophoteric coefficient which ranges in the values between 0.2 and 1.2 and is defined as on using *Rosseland* approximation the energy equations reduce to

$$\vec{u}\frac{\partial T}{\partial y}+\vec{v}\frac{\partial T}{\partial \vec{y}}=\frac{\partial}{\partial \vec{x}}(\alpha\frac{\partial T}{\partial \vec{x}})+\frac{\partial}{\partial \vec{y}}(\alpha\frac{\partial T}{\partial \vec{y}})+\frac{D_BK_T}{C_sC_p}(\frac{\partial^2 C}{\partial \vec{x}^2}+\frac{\partial^2 C}{\partial \vec{y}^2})+\frac{1}{C_p}Q_H+$$
$$+\frac{16\sigma^\bullet T_\infty^3}{C_p\beta_R}(\frac{\partial^2 T}{\partial \vec{x}^2}+\frac{\partial^2 T}{\partial \vec{y}^2}) \quad (7)$$

where $\sigma^\bullet$ is the *Stefan-Boltzmann* constant, β_R is the mean absorption constant.

Introducing the non-dimensional variable

$$x=\frac{\bar{x}}{l},y=\frac{\bar{y}}{l},a=\frac{\bar{a}}{l},\sigma=\frac{\bar{\sigma}}{l},\psi^*=\frac{\bar{\psi}}{l},\theta=\frac{T-T_\infty}{T_w-T_\infty},\ \phi=\frac{C-C\infty}{C_w-C_\infty} \quad (8)$$

the non-dimensional equations in terms of stream function ψ* with variable viscosity and thermal conductivity are

$$(\frac{1}{\theta-\theta_r})(\frac{\partial\theta}{\partial y}\frac{\partial\psi^\bullet}{\partial y}-\frac{\partial\theta}{\partial x}\frac{\partial\psi^\bullet}{\partial x})+(\frac{\partial^2\psi^\bullet}{\partial x^2}+\frac{\partial^2\psi^\bullet}{\partial y^2})+Ra(1-\frac{\theta}{\theta_r})(\frac{\partial\theta}{\partial y}(1+2\gamma_1\theta)$$
$$+N_r\frac{\partial C}{\partial y})-M^2\frac{\partial^2\psi^\bullet}{\partial y^2} \quad (9)$$

$$(\frac{\partial\theta}{\partial x}\frac{\partial\psi^{\bullet}}{\partial y}-\frac{\partial\theta}{\partial y}\frac{\partial\psi^{\bullet}}{\partial x})=(1+\beta\phi+\frac{4Rd}{3})(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2})+\beta((\frac{\partial\theta}{\partial x})^2+(\frac{\partial\theta}{\partial y})^2)+Q \quad (10)$$

$$Le(\frac{\partial\phi}{\partial x}\frac{\partial\psi^{\bullet}}{\partial y}-\frac{\partial\phi}{\partial y}\frac{\partial\psi^{\bullet}}{\partial x})=(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2})-Le\gamma\phi+LeSr((\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\vartheta}{\partial y^2}) \quad (11)$$
$$-Sc\tau(\frac{\partial^2\theta}{\partial x^2}\phi+\frac{\partial^2\theta}{\partial y^2}\phi+\frac{\partial\theta}{\partial x}\frac{\partial\phi}{\partial x}+\frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y})$$

Where $Ra=\frac{\beta_T g(T_w-T_\infty)l}{\alpha_o\nu}$ is the Darcy-Rayleigh number, $\nu=\frac{\mu_\infty}{\rho}$ is the kinematic viscosity of the fluid, $Rd=\frac{4\sigma^* T_\infty^3}{k_f\beta_R}$ is the Radiation parameter, $Q=\frac{Q_H l^2}{(T_w-T_\infty)}$ is heat source parameter, $M^2=\frac{\sigma\mu_e^2 H_0^2 l^2}{\mu}$ .is the magnetic parameter. $Le=\frac{\nu}{D_B}$ is the Lewis number, $\gamma=\frac{kcl^2}{D_B}$ is the chemical reaction parameter, $N=\frac{\beta_c(C_w-C_\infty)}{\beta_0(T_w-T_\infty)}$ is the buoyancy ratio, $Du=\frac{D_B K_T(C_w-C_\infty)}{\alpha_0 C_s C_p(T_w-T_\infty)}$ is the Dufour parameter, $Sr=\frac{D_B K_T(T_w-T_\infty)}{\alpha_0 T_m(C_w-C_\infty)}$ is the Soret parameter, $\tau=-\frac{k}{T_r}(T_w-T_\infty)$ is the thermophoretic parameter and $Sc=\frac{\nu}{D_B}$ is the Schmidt number.In equation(11), $\tau=-\frac{k}{T_r}(T_w-T_\infty)$ represents thermophoresis parameter. From the practical point view two possible cases exist, (i) heated surface $(T_w-T_\infty>0)$, which leads to τ<0, (ii) cold surfaces $(T_w-T_\infty<0)$, which gives rise τ>0.

The transformed boundary conditions are

$$\psi^{\bullet}=0,\theta=1\ ,\phi=1\ \ at\ \ y=aSin(x)$$
$$\frac{\partial\psi^{\bullet}}{\partial y}\to 0,\theta\to 0,\phi\to\infty\ \ as\ \ y\to\infty \quad (12)$$

III. SOULUTION METHODOLOGY

On introducing the following similarity variables as

$$x=\xi,\hat{\eta}=\frac{y-aSin(x)}{\xi^{1/2}Ra^{-1/2}},\psi^{\bullet}=Ra^{1/2}\psi\ ,\ \ \eta=\frac{\bar{\eta}}{(1+a^2Cos^2(\xi))},\psi=\xi^{1/2}f(\eta),\theta=\theta(\eta),,\phi=\phi(\eta)$$

Equations (9)-(11),we obtain a system of ordinary differential equations as follows:

$$f''+(\frac{1}{\theta-\theta_r})\theta' f'-\frac{M^2}{(1+a^2Cos^2\xi)}f''=Ra(1-\frac{\theta}{\theta_r})(\theta'(1+2\gamma\ \theta)+N_r\phi') \quad (13)$$

$$\beta(\theta')^2+\left(1+\beta\theta+\frac{4Rd}{3}\right)\theta''+\frac{1}{2}f\theta'+Q\left(1+a^2Cos^2(\xi)\right)) \quad (14)$$

$$Le\phi''+(\frac{1}{2}Lef-Sc\tau\theta')\phi'-Le\gamma(1+a^2Cos^2(\xi))\phi-Sc\tau\phi\theta''=0 \quad (15)$$

where prime denotes differentiation with respect to η. The corresponding boundary conditions are

$f=0$, θ=1 , ϕ =1 at $\eta=0$ (16)

$f'\to 0$, θ→0, $\phi\to$0 as $\eta\to\infty$

Equations(13)-(15) along with conditions(16) have been solved by fourth order Runge-Kutta shooting technique

In equation (14) the radiation parameter $Rd = \frac{4\sigma^{*}T_{\infty}^{3}}{k_f \beta_R}$ means that the rate of thermal radiation contribution relative to the thermal conditions. As Rd→∞, influence of thermal radiation is high in the boundary layer regime. For Rd→0, the term 4Rd/3 tends to zero. For Rd=1, thermal radiation and thermal conduction will give equal contribution.

The main results of practical interest in many applications are Skin friction coefficient C_f, heat transfer coefficient, and mass transfer coefficient at the surface.
The skin friction coefficient C_f is given by

$$C_f = \frac{f''(0)(1+a^2Cos^2(\xi))Ra^{1/2}}{(1`+M^2+a^2Cos^2(\xi))} \qquad (17)$$

The heat and mass transfer coefficients are expressed in terms of Nusselt and Sherwood numbers Nux, Shx.
Nusselt number Nux and Sherwood number Shx are given by

$$Nux = \frac{xq_w}{\alpha_o(T_w - T_\infty)}, Shx = \frac{xm_w}{D_B(C_w - C_\infty)} \qquad (18)$$

Where q_w is the heat flux on the wavy surface, and is defined by

$q_w = -\alpha_0 \bar{n}.\nabla T \;\; and \;\; \bar{n} = (-\frac{aCos(\xi)}{\sqrt{(1+a^2Cos^2(\xi))}}, \frac{1}{\sqrt{(1+a^2Cos^2(\xi))}})$ is the unit normal vector to the wavy surface, α_0 is the effective porous medium thermal conductivity.Therefore

$$Nu_\xi = -\frac{\theta'(0)Ra_\xi^{1/2}}{\sqrt{(1+a^2Cos^2(\xi))}} \quad , Sh_\xi = -\frac{\phi'(0)Ra_\xi^{1/2}}{\sqrt{1+a^2Cos^2(\xi)}} \qquad (19)$$

IV. RESULTS AND DISCUSSION

From the graphical representations of velocity, temperature, concentration, skin friction, rate of heat and mass transfer we draw the following findings.

Figs.2a-2c depict the variation of velocity, temperature and concentration with magnetic parameter (M). From the profiles we find that the velocity reduces with M in the entire flow region. The temperature and concentration enhance with increasing values of magnetic parameter.This may be attributed to the fact that the thickness of the thermal and solutal boundary layer decay with increasing M.

Figs.3a-3c shows the variation of velocity and temperature with the influence of radiation parameter (Rd). From fig.4a we find that the velocity reduces in the flow region (0, 1.0) and enhances far away from the boundary.This means that the thickness of the momentum boundary layer reduces with increasing values of Rd. Fig.3b&3c represents the temperature and concentration with Rd. It can be seen from the profiles that an increase in Rd leads to thickening of the thermal and solutal boundary layers which results in an enhancement of the temperature and concentration in the flow region.

From 4a-4c represent the effect of thermal conductivity parameter β on the non-dimensional velocity, temperature and concentration. Fig.4a shows the variation of velocity with β.In this case the velocity is found to depreciates in the flow region (0, 1.0) adjacent to the wall and enhances far away from the wall. From fig.4b we found that as the thermal conductivity parameter β increases the temperature enhances.This is due to the thickening of the thermal boundary layer as a result of increasing values of thermal conductivity. From fig.4c, we found that as the thermal conductivity parameter β decreases the concentration reduces in the entire flow region. This is due to the thinning of the solutal boundary layer as a result of increasing values of thermal conductivity.

The effect of thermophoretic parameter (τ) on u, θ and ϕ can be seen from the figs.5a-5c. From fig.5a, we find that increasing the thermophoretic parameter (τ) tends to increase significantly the thickness of the momentum boundary layer Fig.5b reveals that the temperature profile depreciate considerably with increase in τ.From fig.5c it is seen that increase in thermophoretic parameter (τ) leads to raise the values of concentration .In other words increase in τ decelarates the thermal boundary layer thickness while accelerates the solutal boundary layer thickness.

The effect of wall waviness (a) velocity, temperature and concentration can be seen from figs.(6a-6c). From the profile we find that, higher the amplitude (a) of wavy surface larger a velocity, temperature and concentration in the flow region. It may be due to the fact that increasing amplitude (a) growth the thickness of a momentum, thermal and solutal boundary layers.

The skin friction(C_f)(Fig.7a-7b),Nusselt(Fig.8a-8b)) and Sherwood numbers(Fig.9a-9b) (Nu_ξ, Sh_ξ) at the wall η=0 enhance with increasing values of M,Q, β and reduce with viscosity parameter(θr), amplitude of wavy wall(a) at η=0.Higher the radiative heat flux(Rd) reduces $C_{f,}$, Nu_ξ and increases Sh_ξ. $C_{f,}$, Sh_ξ reduces, Nu_ξ enhances with increasing values of thermophoresis parameter(τ) at the wall η

Comparison : With β=0,τ=0,Q=0,Rd=0,M=0 and θr→∞ the present results are in good agreement with *Cheng*[2].

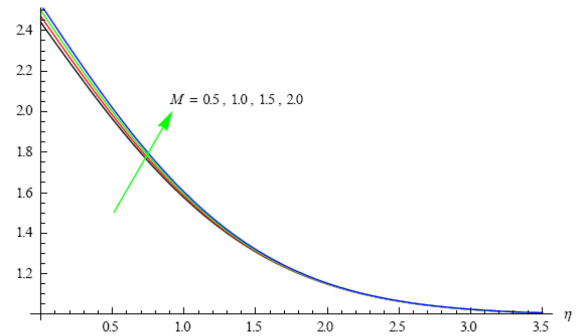


Fig. 2a variation of axial velocity(u) with M
Rd=0.5, β=0.5, τ=0.2, a=0.1

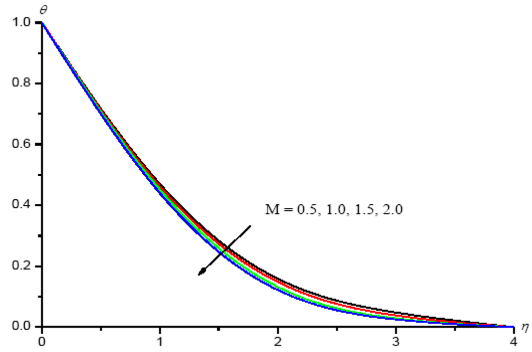


Fig. 2a variation of temperature(θ) with M
Rd=0.5, β=0.5, τ=0.2, a=0.1

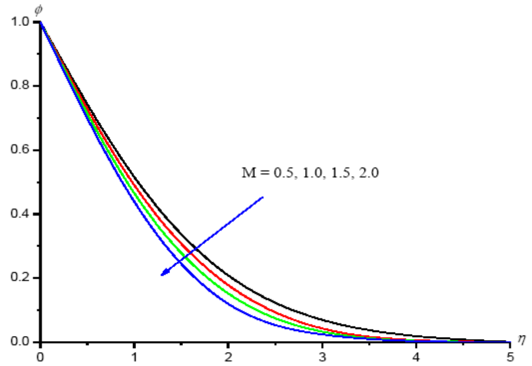


Fig. 2c variation of Concentration (ϕ) with M
Rd=0.5, β=0.5, τ=0.2, a=0.1

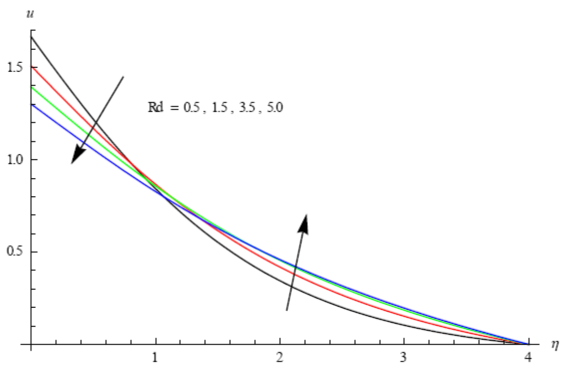


Fig. 3a variation of axial velocity(u) with Rd
M=0.5, β=0.5, τ=0.2, a=0.1

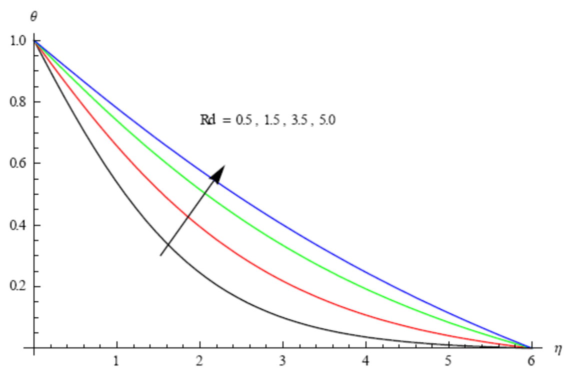


Fig. 3b variation of temperature (θ) with Rd
M=0.5, β=0.5, τ=0.2, a=0.1

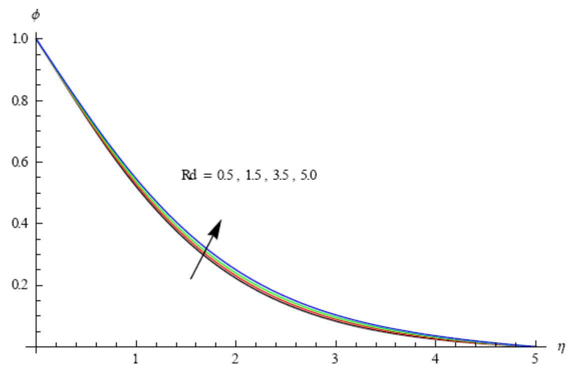


Fig. 3c variation of Concentration (ϕ) with Rd
M=0.5, β=0.5, τ=0.2, a=0.1

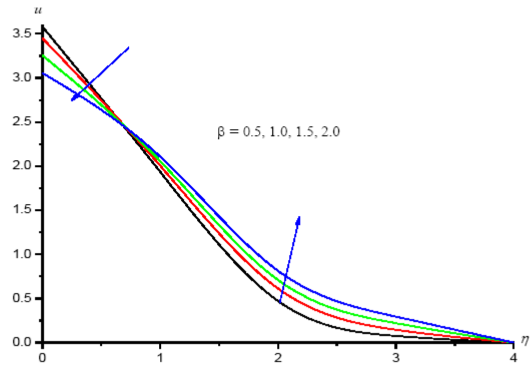


Fig. 4a variation of axial velocity(u) with β
Rd=0.5, τ=0.2, a=0.1, M=0.5

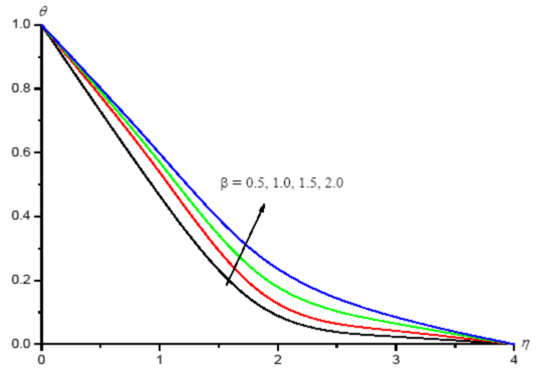


Fig. 4b variation of temperature(θ) with β
Rd=0.5, τ=0.2, a=0.1, M=0.5

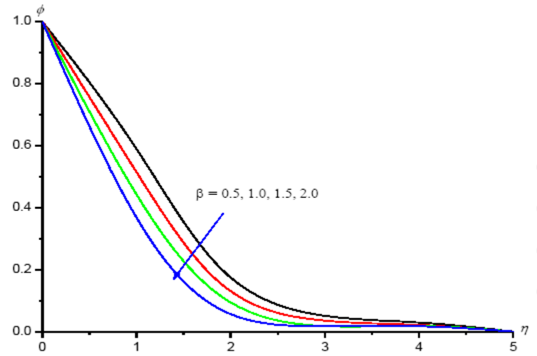


Fig. 4c variation of Concentration (ϕ) with β
Rd=0.5, M=0.5, τ=0.2, a=0.1

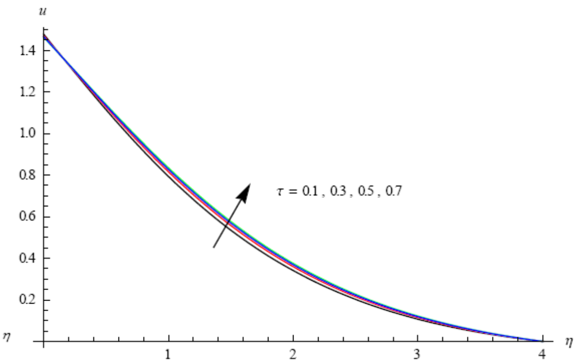


Fig. 5a variation of axial velocity(u) with τ
M=0.5, β=0.5, Rd=0.5, a=0.1

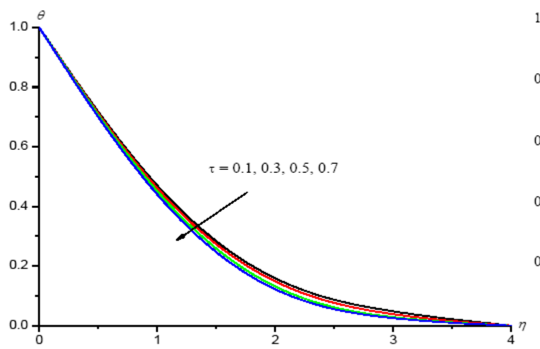


Fig. 5b variation of temperature (θ) with τ
M=0.5, β=0.5, Rd=0.5, a=0.1

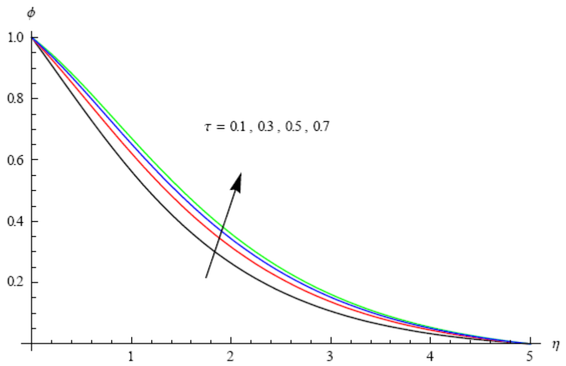


Fig. 5c variation of Concentration (ϕ) with τ
M=0.5, β=0.5, Rd=0.5, a=0.1

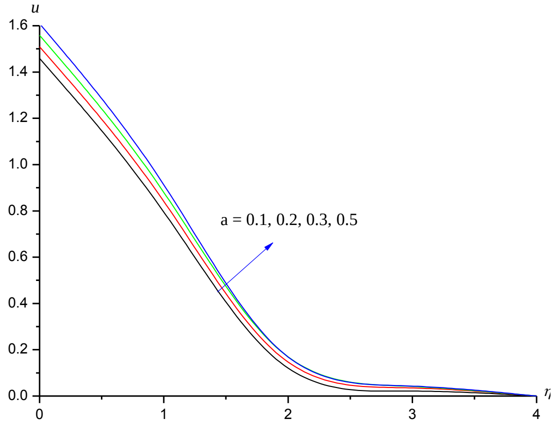


Fig.6a variation of axial velocity(u)with amplitude 'a'
M=0.5,Rd=0.5,β=0.5,θr=-2

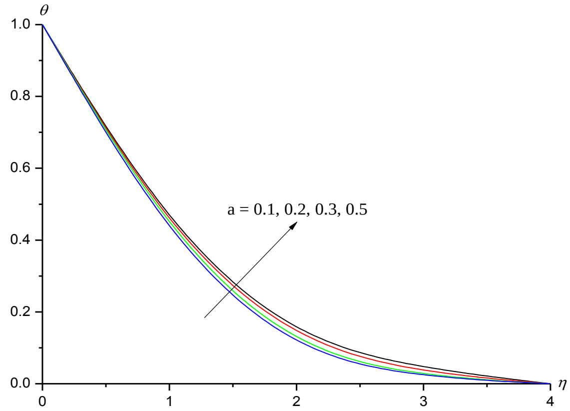


Fig.6b variation of temperature(θ)with amplitude 'a'
M=0.5,Rd=0.5,β=0.5,θr=-2

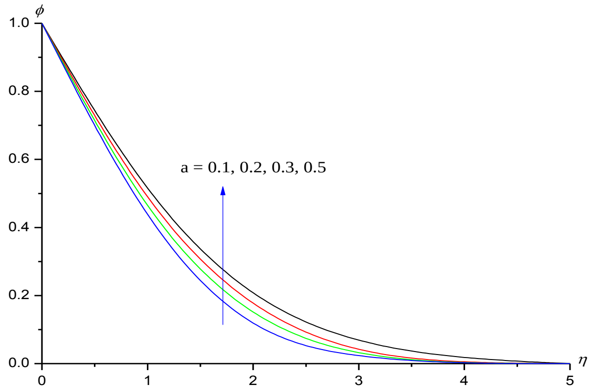


Fig. 6c variation of Concentration(ϕ)with amplitude 'a'
M=0.5,Rd=0.5,β=0.5,θr=-2

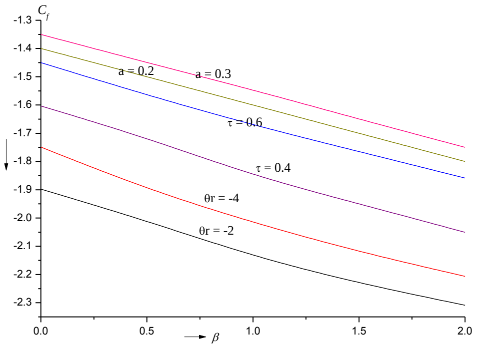


Fig.7a : variation of Skin friction(C_f) with θ_r, τ, a

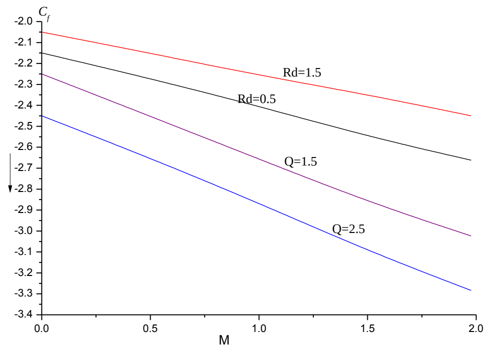


Fig.7b : variation of Skin friction(C_f) with Rd, Q

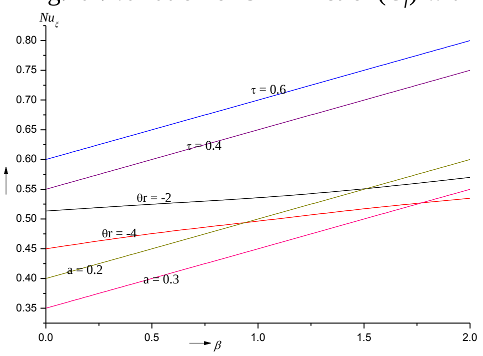


Fig.8a : variation of Nusselt Number (Nu_ξ) with θ_r, τ, a

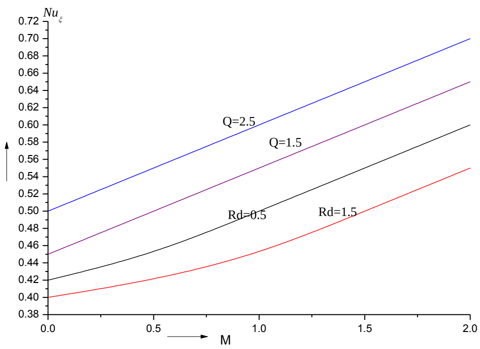
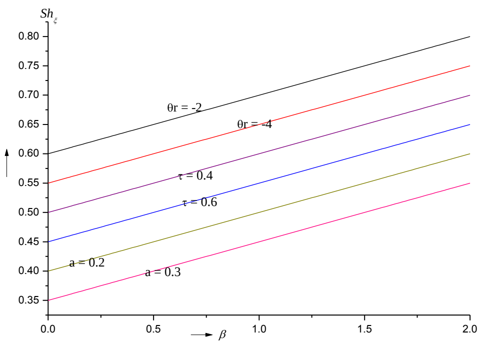


Fig.8b : variation of Nusselt Number (Nu_ξ) with Rd, Q



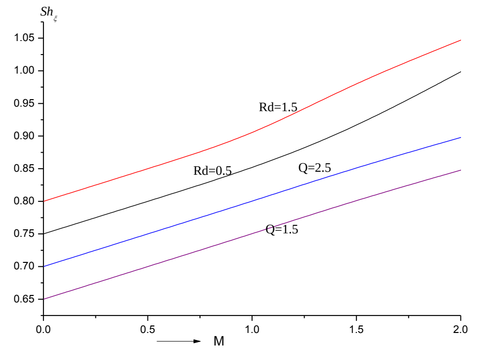


Fig.9a : variation of Sherwood Number (Sh_ξ) with θ_r, τ, a Fig.9b : variation of Sherwood Number (Sh_ξ) with Rd, Q

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